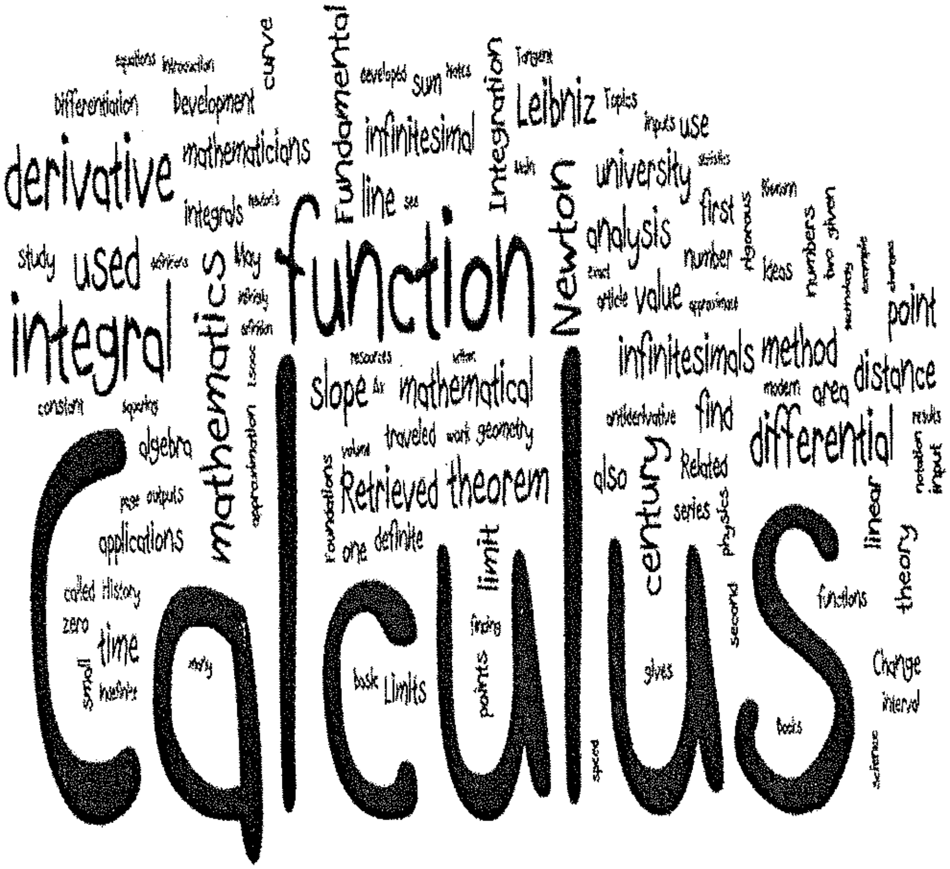


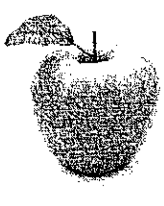
DUE THE FIRST DAY OF SCHOOL

HONORS PRECALC SUMMER ASSIGNMENT

Thank you for signing up for Honors Precalculus. It's a class designed to think deeper about Algebra II/Trig in a way that will prepare you for AP Calc. To that end, attached is your summer assignment that will start to get you thinking about functions. I've included notes for you to review and some follow up questions for you to complete. I look forward to seeing you next year.

VERNON TOWNSHIP MATHEMATICS

Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.
Albert Einstein

 π

Average Rate of Change Function

DEFINITION: A function is a process by which every input is associated with exactly one output. When create a process (or series of steps) to do a certain task we are often creating a function. If we want to use it over and over again then to make our lives easier we give it a name. It helps us remember the name when it has something to do with the process that is being described.

The Average Rate of Change function describes the average rate at which one quanity is changing with respect to something else changing.

You are already familiar with some average rate of change calculations:

(a) Miles per gallon - calculated by dividing the number of miles by the number of gallons used

(b) Cost per killowatt - calculated by dividing the cost of the electricity by the number of killowatts used

(c) Miles per hour - calculated by dividing the numebr of miles traveled by the number of hours it takes to travel them.

In general, an average rate a change function is a process that calculates the the amount of change in one item divided by the corresponding amount of change in another. Using function notation, we can define the Average rate of Change of a function f from a to x as

$$A(x)=\frac{f(x)-f(a)}{x-a}$$

- A is the name of this average rate of change function
- x - a represents the change in the input of the function f
- f(x) - f(a) represents the change in the function f as the input changes from a to x

You might have noticed that the Average Rate of Change function looks a lot like the formula for the slope of a line. In fact, if you take any two distinct points on a curve, (x_1,y_1) and (x_2,y_2), the slope of the line connecting the points will be the average rate of change from x_1 to x_2

Example 1: Find the slope of the line going through the curve $f(x)=\frac{1}{3}x^2-4$ as x changes from 3 to 0.

Step 1: f (3) = -1 and f (0) = -4

Step 2: Use the slope formula to create the ratio

$$\frac{f(0)-f(3)}{0-3}$$

Step 3: Simplify.

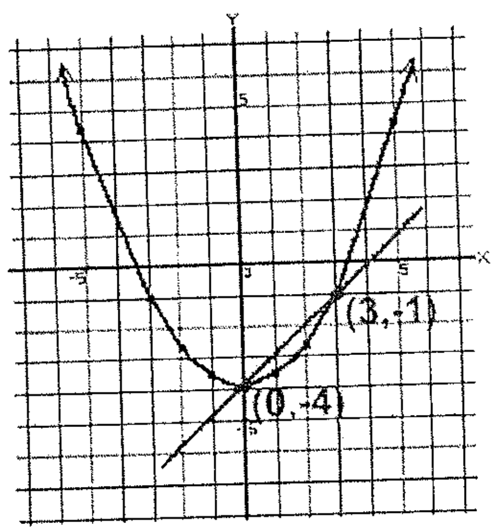
$$\frac{f(0)-f(3)}{0-3}=\frac{-4-(-1)}{0-3}$$
$$=1$$

Step 4: So the slope of the line going through the curve $f(x)=\frac{1}{3}x^2-4$ as x changes from 3 to 0 is 1.

Example 2: Find the average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 0.

Since the average rate of change of a function is the slope of the associated line we have already done the work in the last problem. That is, the average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 0 is 1. That is, over the interval [0,3], for every 1 unit change in x, there is a 1 unit change in the value of the function.

Here is a graph of the function, the two points used, and the line connecting those two points.



Now suppose you needed to find series of slopes of lines that go through the curve and the point (3, f(3)) but the other point keeps moving. We will call the second point (x, f(x)). It will be useful to have a process (function) that will do just that for us. The average rate of change function also deterines slope so that process is what we will use.

Example 3: Find the average rate of change function of $f(x)=\frac{1}{3}x^2-4$ from 3 to x.

Step 1: f (3) = -1 and $f(x)=\frac{1}{3}x^2-4$

Step 2: Use the average rate of change formula to define A(x) and simplify.

$$A(x)=\frac{f(x)-f(3)}{x-3}$$
$$=\frac{\left(\frac{1}{3}x^2-4\right)-(-1)}{x-3}$$
$$=\frac{\frac{1}{3}x^2-3}{x-3}$$
$$=\frac{x^2-9}{3(x-3)}$$
$$=\frac{(x-3)(x+3)}{3(x-3)}$$
$$=\frac{(x+3)}{3}, x\neq 3$$

Step 3: The average rate function of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to x is $A(x)=\frac{(x+3)}{3}, x\neq 3$

Example 4: Use the result of Example 3 to find the average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 6.

Solution: The average rate function of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to x is $A(x)=\frac{(x+3)}{3}, x\neq 3$

So, the average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 6 is A(6) = 9/3 = 3.

Example 5: Use the result of Example 3 to find the average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 0.

The average rate of change of $f(x)=\frac{1}{3}x^2-4$ from 3 to 0 is A(0) = 3/3 = 1.

Name ______________________________ Date ______________

Word problems involving rate of change

1. When the dependent variable increases when the independent variable increases, the rate of change is (Positive, negative, zero, undefined) circle one.

2. When the dependent variable stays the same as the independent variable increases, the rate of change is (Positive, negative, zero, undefined) circle one.

3. When the dependent variable decreases as the independent variable increase, the rate of change is (Positive, negative, zero, undefined) circle one.

4. When the dependent variable increase as the independent variable stays the same, the rate of change is (Positive, negative, zero, undefined) circle one.

Find the rate of change
(Hint: word problems are $\frac{\text{units}}{\text{time}}$ Identify what you are given and determine the unit and the time.)
Write the ordered pair (time, units).

5.

X	Y
20	35
25	40

6. A climber is on a hike. After 2 hours he is at an altitude of 400 feet. After 6 hours, he is at an altitude of 700 feet. What is the average rate of change?

7. A scuba diver is 30 feet below the surface of the water 10 seconds after he entered the water and 100 feet below the surface after 40 seconds. What is the scuba divers rate of change?

8. A rocket is 1 mile above the earth in 30 **seconds** and 5 miles above the earth in 2.5 **minutes.** What is the rockets rate of change in miles per second? What about miles per minute.

9. A teacher weighed 145 lbs in 1986 and weighs 190 lbs in 2007. What was the rate of change in weight?

10. Over the last 50 years, the average temperature has increased by 2.5 degrees worldwide (I made this up). What is the rate of change in worldwide temperatures per year?

11. Michael started a savings account with $300. After 4 weeks, he had $350 dollars, and after 9 weeks, he had $400. What is the rate of change of money in his savings account per week?

12. A plane left Chicago at 8:00 A.M. At 1: P.M., the plane landed in Los Angeles, which is 1500 miles away. What was the average speed of the plane for the trip?

13. After 30 baseball games, A-Rod had 25 hits. If after 100 games he had 80 hits, what is his average hits per baseball game.

Find the slope of a line that has these points

14. (8,2) and (11,3)

15. (8,0) and (8, 6)

Transforming Parent Functions

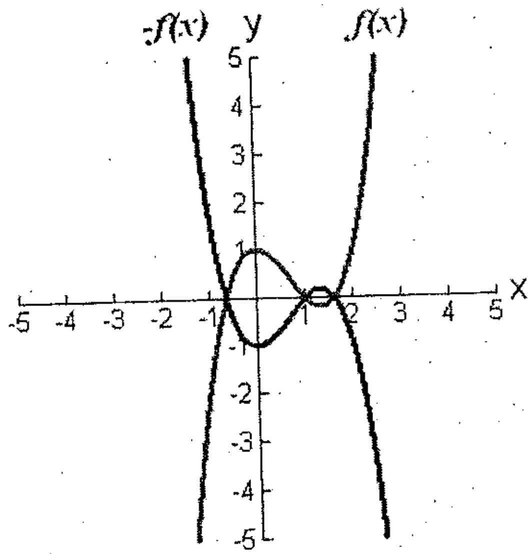
Reflections and Functions: Examining $-f(x)$ and $f(-x)$

Reflection over the x-axis

$-f(x)$ reflects $f(x)$ over the x-axis.

A **reflection** is a mirror image. Placing the edge of a mirror on the x-axis will form a reflection in the x-axis. This can also be thought of as "folding" over the x-axis.

If the original (parent) function is $y = f(x)$, the**reflection over the x-axis** is function $-f(x)$.

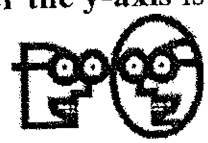


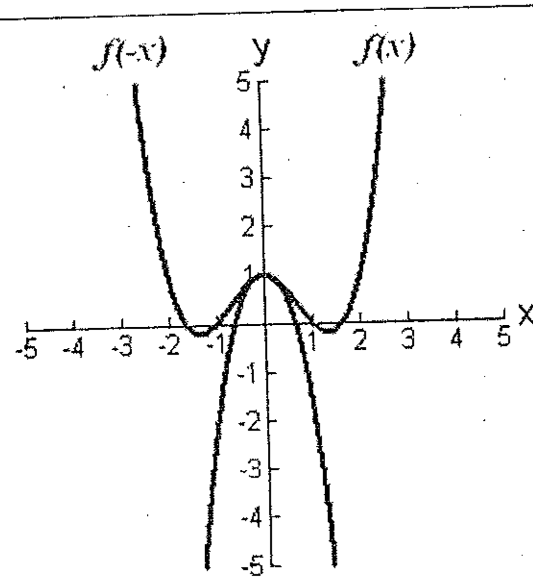
Reflection over the y-axis

$f(-x)$ reflects $f(x)$ over the y-axis.

Placing the edge of a mirror on the y-axis will form a reflection in the y-axis. This can also be thought of as "folding" over the y-axis.

If the original (parent) function is $y = f(x)$, the**reflection over the y-axis** is function $f(-x)$.





Translations and Functions: Examining $f(x + a)$ and $f(x) + a$

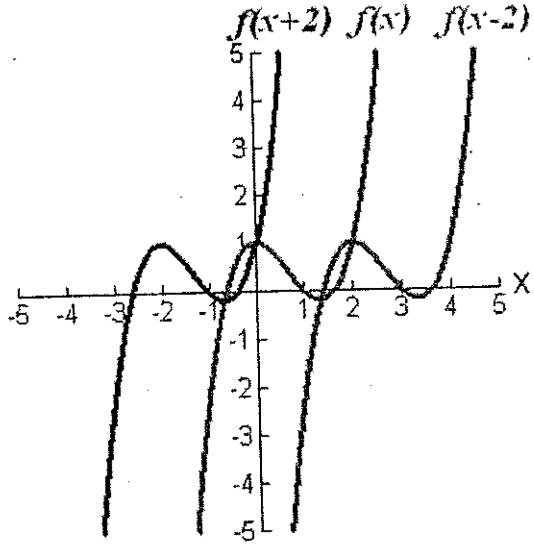
Slide to the right or left

$f(x + a)$ translates $f(x)$ horizontally

If the original (parent) function is $y = f(x)$, the translation (sliding) of the function horizontally to the left or right is given by the function $f(x - a)$.

- if $a > 0$, the graph translates (slides) to the**right**.
- if $a < 0$, the graph translates (slides) to the**left.**

Remember that you are "subtracting" the value of a from x. Thus $f(x + 2)$ is really $f(x - (-2))$ and the graph moves to the **left.**



$f(x) = x^3 - 2x^2 + 1$

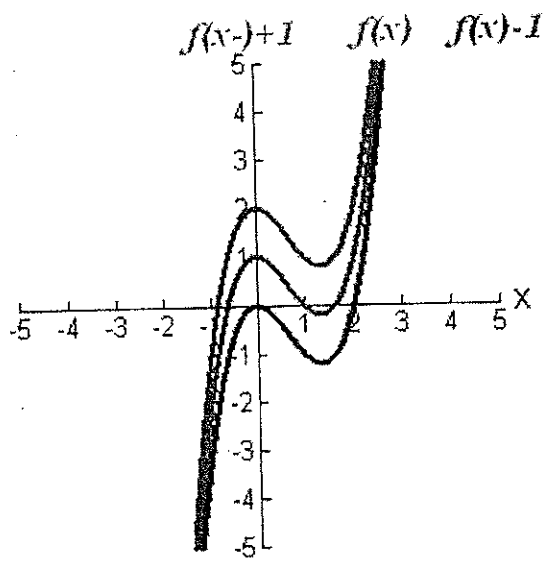
Slide upward or downward

$f(x) + a$ translates $f(x)$ vertically

If the original (parent) function is $y = f(x)$, the translation (sliding) of the function vertically upward or downward is the function $f(x) + a$.

- if $a > 0$, the graph translates (slides) **upward.**
- if $a < 0$, the graph translates (slides) **downward.**

Remember that you are adding the value of a to the$y$-values of the function.

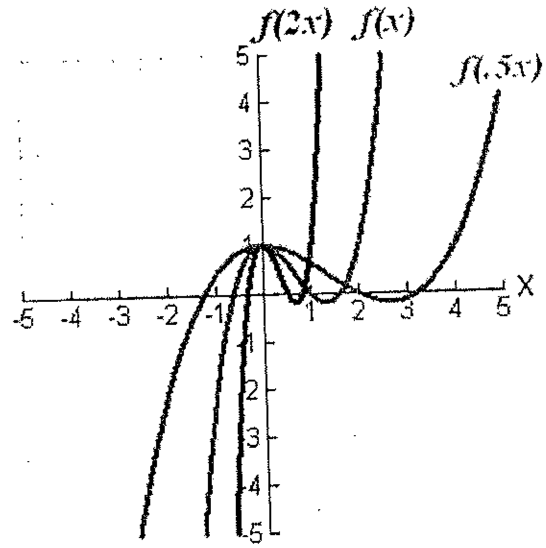


$f(x) = x^3 - 2x^2 + 1$

Stretch or Compress Functions: Examining $f(ax)$ and $a\,f(x)$

Horizontal Stretch or Compress

$f(ax)$ stretches/compresses $f(x)$ horizontally



$f(x) = x^3 - 2x^2 + 1$

A **horizontal stretching** is the stretching of the graph away from the *y*-axis.
A **horizontal compression** is the squeezing of the graph towards the *y*-axis.

If the original (parent) function is $y = f(x)$, the horizontal stretching or compressing of the function is the function $f(ax)$.

- if $0 < a < 1$ **(a fraction)**, the graph isstretched **horizontally** by a factor of a units.
- if $a > 1$, the graph is **compressed horizontally** by a factor of a units.
- if ***a* should be negative**, the horizontal compression or horizontal stretching of the graph is followed by a reflection of the graph across the *y*-axis.

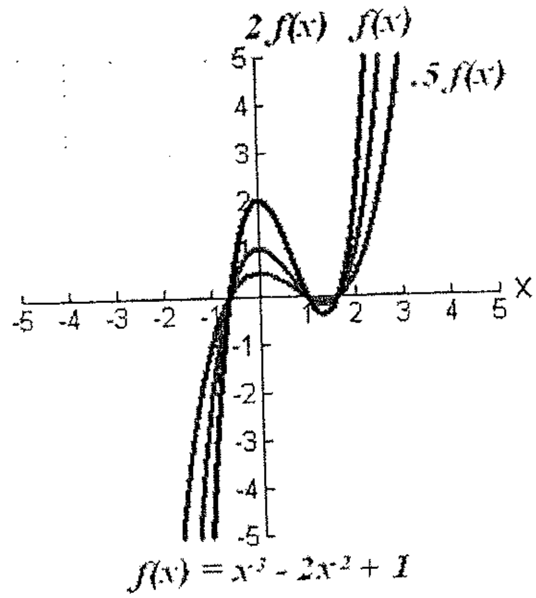
Vertical Stretch or Compress

$a\,f(x)$ stretches/compresses $f(x)$ vertically

A **vertical stretching** is the stretching of the graph away from the x-axis.
A **vertical compression** is the squeezing of the graph towards the x-axis.

If the original (parent) function is $y = f(x)$, the vertical stretching or compressing of the function is the function $a\,f(x)$.

- if $0 < a < 1$ **(a fraction)**, the graph is**compressed vertically** by a factor of a units.
- if $a > 1$, the graph is **stretched vertically** by a factor of a units.



- If a **should be negative,** then the vertical compression or vertical stretching of the graph is followed by a reflection across the x-axis.

Algebra II: Translations on Parent Functions Review

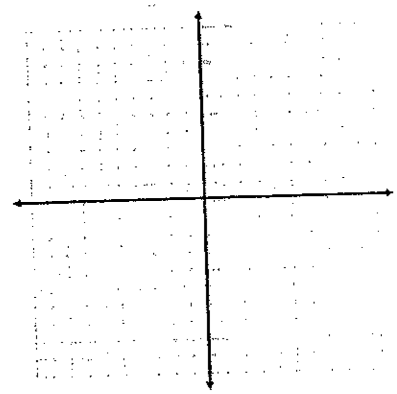
Name ______________________________ Period ______ Date ____________

For problem 1- 6, please give the name of the parent function and describe the transformation represented. You may use your graphing calculator to compare & sketch.

1. $g(x) = x^2 - 6$

Parent: ____________________

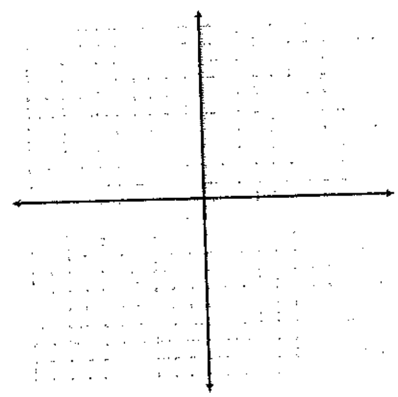
Transformations: ____________________



2. $f(x) = |x-1|$

Parent: ____________________

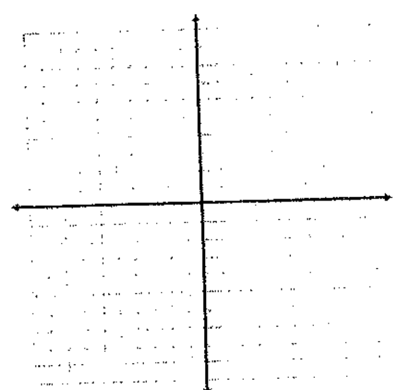
Transformations: ____________________



3. $h(x) = \sqrt{x} + 4$

Parent: ____________________

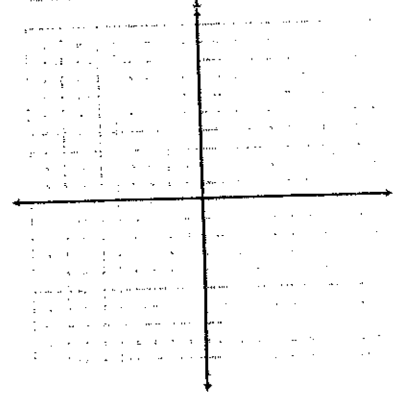
Transformations: ____________________



4. $g(x) = (x+1)^2 + 3$

Parent: ____________________

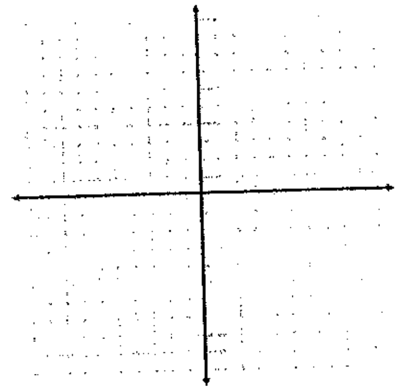
Transformations: ____________________



5. $g(x) = x-2$

Parent: ______________________

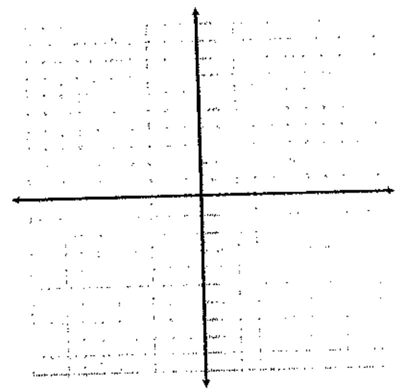
Transformations: ______________________



6. $f(x) = |x+5| - 2$

Parent: ______________________

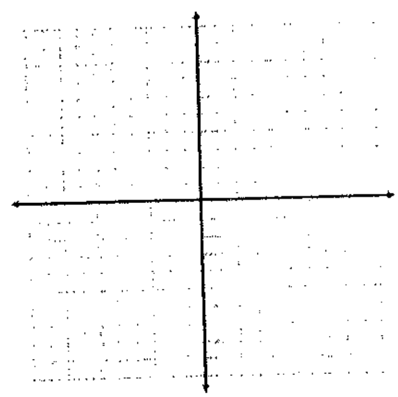
Transformations: ______________________



7. $h(x) = \sqrt{x+2} - 5$

Parent: ______________________

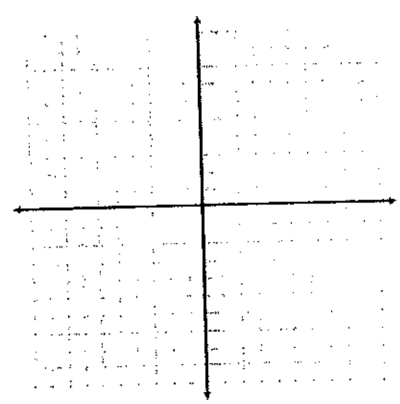
Transformations: ______________________



8. $h(x) = x^2 + 1$

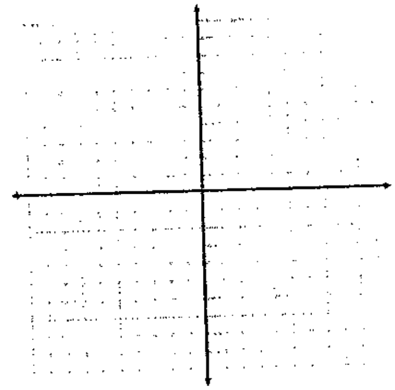
Parent: ______________________

Transformations: ______________________



9. h(x) = $x^3 - 2$ Parent: ______________________

Transformations: ______________________



For problems 10 – 14, given the parent function and a description of the transformation, write the equation of the transformed function, f(x).

10. Absolute value—vertical shift down 5, horizontal shift right 3. ______________________

11. Linear—vertical shift up 5. ______________________

12. Square Root —vertical shift down 2, horizontal shift left 7. ______________________

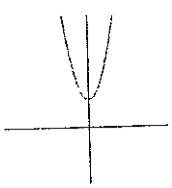
13. Quadratic— horizontal shift left 8. ______________________

14. Quadratic—vertex at (-5, -2). ______________________

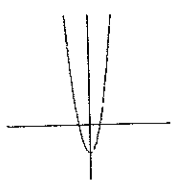
For problems 15 & 16, circle the graph that best represents the given function.

15. f(x) = x^2 - 2?

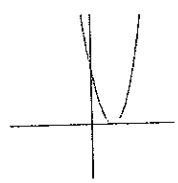
a.



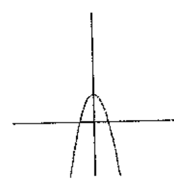
b.



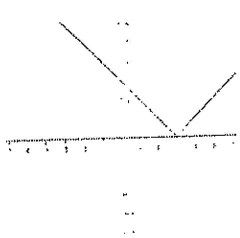
c.



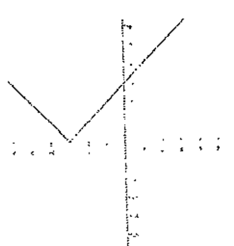
d.



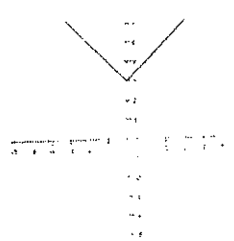
16. g(x) = |x+3|?



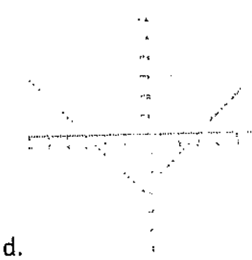
a.



b.



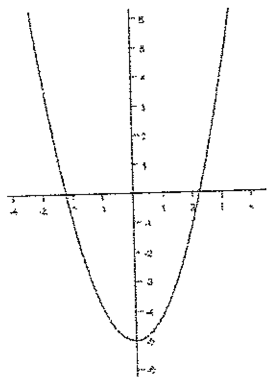
c.



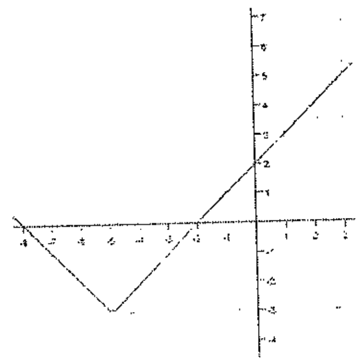
d.

Write the equation for the following translations of their particular parent graphs. You may use y= or function notation (the f(x) type notation).

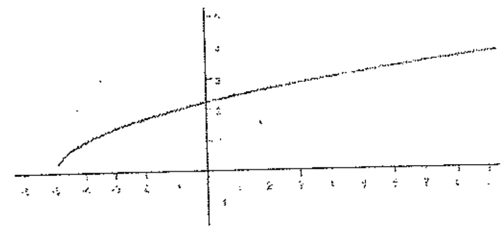
17.________________________



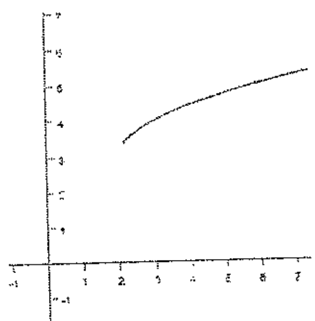
18 .________________________



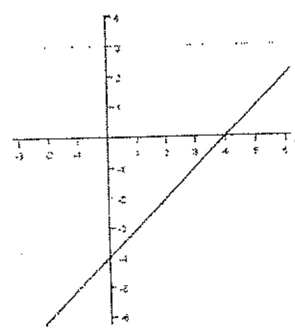
19. ________________________



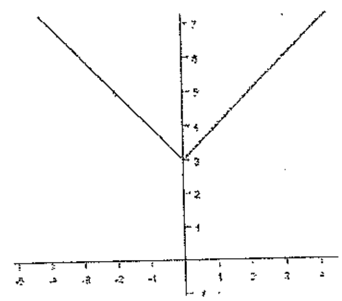
20. ________________________



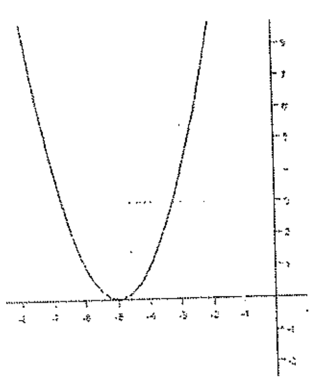
21. ________________________



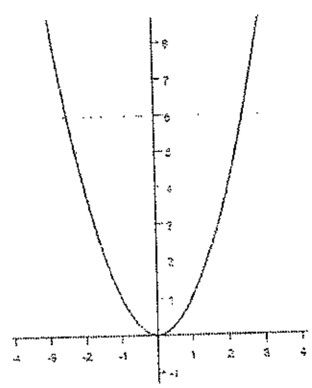
22. ________________________



23. ________________________



24. ________________________



25. ________________________